Case Study: Model to Simulate Regional Flow in South Florida

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Abstract: South Florida has a complex regional hydrologic system that consists of thousands of miles of networked canals, sloughs, highly pervious aquifers, open areas subjected to overland flow and sheet flow, agricultural areas and rapidly growing urban areas. This region faces equally complex problems related to water supply, flood control, and water quality management. Advanced computational methods and super fast computers alone have limited success in solving modern day problems such as these because the challenge is to model the complexity of the hydrologic system, while maintaining computational efficiency and acceptable levels of numerical errors. A new, physically based hydrologic model for South Florida called the regional simulation model (RSM) is presented here. The RSM is based on object oriented design methods, advanced computational techniques, extensible markup language, and geographic information system. The RSM uses a finite volume method to simulate two-dimensional (2D) surface and groundwater flow. It is capable of working with unstructured triangular and rectangular mesh discretizations. The discretized control volumes for 2D flow, canal flow and lake flow are treated as abstract "water bodies" that are connected by abstract "water movers." The numerical procedure is designed to work with these and many other abstractions. An object oriented code design is used to provide robust and highly extensible software architecture. A weighted implicit numerical method is used to keep the model fully integrated and stable. A limited error analysis was carried out and the results were compared with analytical error estimates. The paper describes an application of the model to the L-8 basin in South Florida and the strength of this approach in developing models over complex areas.

Introduction

South Florida is a very complex hydrologic system. Its complexity is mainly due to the considerable groundwater and surface–water interaction, spatial variability in land use, hundreds of flow control structures, extensive wetlands systems, adjacent urban areas, influence of Lake Okeechobee, and the unique flow characteristics of the Everglades and the water conservation areas. Even if the computing power is continuously increasing every year, the complexity of the hydrologic system and the water management issues in South Florida have been increasing at an even faster rate. Consequently, more efficient computational methods, more flexible computer codes, better code development environments, and better code maintenance procedures are needed to keep pace with these growing demands. The need for clean code design, participation by multiple developers from a variety of

disciplines, and regular use of test cases to routinely check code integrity has become critical. Application of a number of new technologies described below has contributed to resolve some of these problems.

The first technological contribution came from recent developments in information technology and the use of object oriented (OO) code design methods. The use of extensible markup language (XML) (Bosak and Bray 1999), geographic information system (GIS) technology, and database support has allowed us to achieve a level of code flexibility and data integration that did not exist before. Object oriented methods have been used in the past for hydraulic model design by Solomontine (1996), Tisdale (1996), and many others. Although OO design may have been previously considered to be outside the expertise of many hydrologists, the increased complexity of the hydrologic processes involved, and the need to incorporate methods developed by professionals from many disciplines such as biology, hydrogeology, and ecology have changed this view. The strong dependencies between hydrology, nutrient transport, and ecology have created a need to integrate various approaches and therefore to integrate computer codes. Simple models that address issues within one discipline at a time have become inadequate for studying complex systems. However, the improved use of GIS support tools, OO code design, and XML language have made it possible to model complex systems, and organize and present large amounts of complex data.

The second technological contribution came from developments in computational methods. Use of unstructured meshes of variable size to simulate two-dimensional (2D) integrated overland and groundwater flow in irregular shaped domains has become common. Full and partial integration with canal networks

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and lakes is now possible. In the past 2 decades, a number of physically based, distributed-parameter models have emerged with such features. The early models include *MODBRANCH* by Swain and Wexler (1996), *MODNET* by Walton et al. (1999), *Mike SHE* based on Abbott et al. (1986a,b), *WASH123* by Yeh et al. (1998), *MODFLOW-HMS* by HydroGeoLogic (2000), and models by VanderKwaak (1999), Schmidt and Roig (1997), and Lal (1998). The computational engines of these models are based on solving a form of the shallow water equation for surface flow and either the variably saturated Richards' equation or the fully saturated groundwater flow equation. Inertia terms in the shallow water equations were neglected, and the solution to the governing equations was obtained using a single global matrix. A number of features are available in these models to simulate structures, urban areas, and agricultural areas. The choice of features depends on the intended application of the model.

Some developments in numerical error analysis by Hirsch (1989) and Lal (2000) also helped in the selection of optimal discretizations for integrated models. Results of error analysis are useful in developing model meshes that produce more accurate solutions and avoid large errors and incipient instabilities. Large-scale integration using implicit methods is practically impossible without understanding numerical error and instability. Because of unconditional stability, implicit models can be run with practically any time step regardless of whether the solution is accurate or not.

The third contribution came from a new generation of computer packages that can be used to solve large sparse systems of equations efficiently. It is now possible to develop implicit finite volume algorithms and solve many complex equations simultaneously without iterating between various model components. Modern solvers such as *PETSC* (Balay et al. 2001) support parallel processing, and have a variety of built-in tools and options to achieve fast model runs. These solvers are easy to use because details such as matrix storage methods are hidden from the user. The current model uses the software package *PETSC* (Balay et al. 2001) to solve the matrices.

The most commonly used integrated model in South Florida is the South Florida Water Management Model (SFWMM) (SFWMD 1999). This model has been adopted to simulate regional hydrology and water management practices since the late 1970s. It simulates the water resources system from Lake Okeechobee in the north to Florida Bay in the south, covering an area of 19,700 km² with a mesh of 3.22 km by 3.22 km (2 mi by 2 mi) cells. The model simulates the major components of the hydrologic cycle including rainfall, evapotranspiration, overland and groundwater flows, canal flow, canal seepage, levee seepage, and well pumping. It incorporates current or proposed water management protocols and operational rules. The success of the model has resulted in an increased demand for its use along with a growth of its size and complexity beyond what was originally intended. The code gradually became very complex, difficult to understand, improve, and expand. This was the primary factor that motivated the launching of a "new generation" regional simulation model (RSM). Unlike the SFWMM that was written in *FORTRAN*, the RSM code is being developed using an OO design and the *C++* language. These choices were made so that the code design can allow for easy modification, growth, and participation of multiple developers.

The RSM is functionally a combination of a hydrologic simulation engine (HSE), which executes the flow simulations, and a management simulation engine (MSE) that can represent structure and pump operations. The HSE has been used to simulate flow in

the Kissimmee River by Lal (1998), and in the Everglades National Park by Lal et al. (1998), and Brion et al. (2000, 2001). The accuracy of the model was verified using the *MODFLOW* model and an analytical solution for stream–aquifer interaction (Lal 2001). This paper describes the model formulation and the object oriented design of the HSE. This model makes it possible to fully integrate the components of the system, and allow for expansion using new hydraulic components, land use types, microhydrological features, canals, reservoirs, and other aspects of the system. An error analysis was carried out to determine the relationship of the numerical error to the size of the triangular cells and the time step. The results of this analysis can be useful in the design of spatial and temporal discretizations to minimize numerical error.

The paper includes an HSE application example that simulates the hydrology in the L-8 basin of South Florida. The L-8 basin is a relatively simple basin in South Florida that has some of the complexities of the regional hydrologic system. Many of these complexities apply to conditions outside South Florida as well. The results of the simulation example are used to demonstrate why this approach was chosen to develop the new RSM.

Governing Equations

The governing equations for the integrated overland–groundwater–canal–lake flow system consist of mass balance or continuity equations and equations of motion. For overland and canal flow, noninertia form of the Saint Venant equation is used as the governing equation. All the governing equations written in conservative form are finally assembled together in the implicit implementation of the finite volume method.

Overland and Groundwater Flow

The 2D continuity equation from St Venant equations for unsteady overland flow and unsteady saturated groundwater flow in a single layered aquifer can be expressed using

$$s_c \frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} + \frac{\partial (vh)}{\partial y} - R_{rchg} + W = 0 \qquad (1)$$

in which u and $v =$ flow velocities in the x and y directions; $h =$ water depth for overland flow and saturated aquifer thickness for groundwater flow; $R_{rchg} =$ net contribution of the recharge from local hydrology into the regional system; $W =$ source or sink terms, for example, due to pumping wells measured in units of volume rate per unit area or meters per second; $s_c =$ storage coefficient; and $s_c = 1$ for overland flow. The term R_{rchg} is also measured in units of meters per second, and is computed for discretized cells referred to as pseudocells designed to capture the recharge from local hydrology. A number of pseudo cell models are described later.

When inertia terms are neglected, the momentum equation reduces to the diffusion flow equation. Diffusion flow assumption is valid for overland flow when inertia terms are small. Akan and Yen (1981) and Hromadka et al. (1987) and many others have used diffusion flow models for many practical applications. Ponce et al. (1978) found the condition of applicability for these models as $T_p S_0 \sqrt{g/h} > 30$ in which $T_p =$ period of the smallest sinusoidal disturbance in the solution; $S_0 =$ bed slope. Lal (2000) found that this condition applies for most regional flows in South Florida

with $T_p > 4$ days. It can be shown that the momentum equations under the diffusion flow assumption and the equations describing Darcy's law can be written as

$$u = -\frac{T(H)}{h}\frac{\partial H}{\partial x}$$

$$v = -\frac{T(H)}{h}\frac{\partial H}{\partial y} \tag{2}$$

in which u, v = average flow velocities in x and y directions; and H = water head. For overland flow, $H = h + z$; h = water depth; z = ground elevation. For groundwater flow, h = saturated aquifer thickness; T = transmissivity of the aquifer. For both overland and groundwater flows, $T(H)$ can be expressed as a function of the state variable H. For single layered groundwater flow, $T = k_h(H - z_b)$ in which z_b = elevation of aquifer bottom; and k_h = hydraulic conductivity of the aquifer. For overland flow, $T = C(H)|S_n|^{\lambda-1}$ in which $C(H)$ is defined as the conveyance where S_n = magnitude of the maximum water surface slope which is approximately equal to S_f, the energy slope under the diffusion flow assumption (Akan and Yen 1981). Variable λ = an empirical constant described later. The purpose of keeping generic functions for $T(H)$ and $C(H)$ is to use object oriented design methods and allow for the implementation of a variety of flow behaviors. These functions can represent constants, analytic functions or lookup tables based on field experiments. Abstract representations of $T(H)$ and $C(H)$ can be used to describe complex flow through wetlands. A power function that can describe many flow resistance equations including the Manning equation, the laminar flow equation, and a number of other wetland equations is $V_n = (1/n_b)h^\gamma S_n^\lambda$, in which V_n = average flow velocity. This equation can be used to derive an expression for $T(H)$ as

$$T(H) = \frac{(H-z)^{\gamma+1}|S_n|^{\lambda-1}}{n_b} \tag{3}$$

in which $S_n = \mathrm{Max}(S_n, \delta_n)$ is used when $\lambda < 1$ to prevent division by zero at $S_n = 0$ and $H > z$. The discharge per unit width can be described now as $q(H) = T(H)S = C(H)|S|^{\lambda-1}S$. The variable $\delta_n = 10^{-13} - 10^{-7}$ is used in the flat terrains of South Florida. A large value of δ_n allows more flat areas of the system to be solved by an approximate form of the Manning equation and prevent instability. Eq. (3) can also be used in wetlands by selecting the parameters suggested by Kadlec and Knight (1996). For the Manning equation, $\gamma = 2/3$; $\lambda = 1/2$; and n_b = Manning constant. Comprehensive flow equations for shallow streams have recently been developed by Katul (2002) and Lopez and Garcia (2001). The abstract base class for $C(H)$ allows for such functions to be seamlessly accommodated in to the model.

Canal Flow

The 1D St Venant equations are used to describe gradually varied unsteady canal flow. The continuity equation for canal flow in conservative form is

$$\frac{\partial A_c}{\partial t} + \frac{\partial Q}{\partial n} - R_{\text{canal}} + W = 0 \tag{4}$$

in which A_c = cross sectional area of the canal; Q = discharge rate; n = distance along the canal; R_{canal} = volume rate at which water is entering the canal due to seepage and other sources per unit length (L^2/T); and W = source and sink terms due to pumps. When diffusion flow is assumed and inertia terms are neglected,

gravity and friction terms left in the momentum equation give $Q = C(R)/\sqrt{|S_n|}S_n = A_c R^{2/3}/(\sqrt{|S_n|}n_b)S_n$, in which R = hydraulic radius; S_n = water surface slope; and $S_n = S_f$ = friction slope as assumed by Akan and Yen (1981). The purpose of writing the equation in this form is to create a generic function $C(R)$ for conveyance that is not limited to use the Manning equation. Discharge Q can now be expressed as

$$Q = -T_c\frac{\partial H_c}{\partial n} \tag{5}$$

in which H_c = canal water level; $T_c = A_c R^{2/3}/(n_b\sqrt{|S_n|})$; and $C(R) = A_c R^{2/3}/n_b$ when the Manning equation is used. Slope S_n is computed using $\mathrm{Max}(S_n, \delta_n)$ with $\delta_n = 10^{-13} - 10^{-7}$ as described earlier.

Lake Flow

The equation governing mass balance in a lake is

$$A_l\frac{\partial H_l}{\partial t} - R_{\text{lake}} + W = 0 \tag{6}$$

in which A_l = lake area; H_l = water level; and R_{lake} = net volume rate at which water is entering the lake water body due to leakage. The governing equations written in conservative form are used in the implicit implementation of the finite volume method.

Recharge from Local Hydrologic System

The local hydrology in a regional system depends on the local land use type along with all its management practices. Different land use types generate different recharges and therefore different hydrologic responses. The recharge R_{rchg} described in Eq. (1) therefore has to be computed separately for each cell with a new land use type. The computations take into account evapotranspiration (ET), rainfall, soil moisture effects, urban detention, local drainage effects, agricultural practices, and local management practices depending on the land use type. The equation of mass balance is used to compute recharge as

$$R_{\text{rchg}} = P - E + I - \frac{dU_s}{dt} - \frac{dD}{dt} \tag{7}$$

in which R_{rchg} = recharge recharge rate (m/s) computed as volume rate per unit cell area entering into the cell; P = precipitation rate; E = evapotranspiration rate; I = water entering the cell during irrigation and other similar functions; U_s = unsaturated moisture depth; and D = detention volume converted to depth. The rates dU_s/dt and dD/dt if used, depend on infiltration and percolation rates of the local cell. In the model, these complex computations are carried out within the pseudocell (object) of each respective cell. Pseudo cells are developed for various land use types, permitting conditions or management practices. More information about pseudo cells is provided under the object design.

Implicit Finite Volume Method

Governing equations for overland flow, groundwater flow, canal flow, lake flow, and other types of flow are based on conservation laws and can be solved using the finite volume method. Eqs. (1), (4), and (6) can be written in the form

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} + \frac{\partial G(U)}{\partial y} + S(U) = 0 \qquad (8)$$

in which U = conservative variable representing H, H_c, or H_l; variables $F(U)$ and $G(U)$ = the x and y components of flux in 2D flow, and $S(U) = -R_{rchg} + W$ = summation of source/sink terms. The finite volume formulation is applied to all 2D, 1D, and lake related regional flows. The numerical model is developed for generic control volumes and is applied to all water bodies on an equal basis regardless of whether they are 2D cells, canal segments, or lakes.

The finite volume formulation for the governing Eq. (8) is derived by integrating it over an arbitrary control volume or water body Ω

$$\frac{\partial}{\partial t} \int_\Omega U d\Omega + \int_S (\mathbf{E} \cdot \mathbf{n}) dA + \int_\Omega S d\Omega = 0 \qquad (9)$$

in which $\mathbf{E} = [F, G]^T$ = flux rate across the wall and $\mathbf{n}$ = unit vector normal to the wall. The first term of Eq. (9) represents the rate of change of water volumes in water bodies such as cells, canal segments, and lakes. The second term is obtained using the Gauss' theorem and contains the sum of fluxes crossing the control surfaces of the water bodies. This term contains all flow exchanges among the water bodies. Any mechanism that is capable of moving water between any two water bodies is defined as an abstract water mover. Water bodies and water movers are two of the basic building blocks of the model. They eventually become abstract base classes in the OO design. These abstractions are capable of growing and evolving into various model objects as the model evolves.

In the model, Eq. (9) is solved for average water heads H, H_c, or H_l of all the water bodies simultaneously. In the finite volume formulation, Eq. (9) reduces to the following system of differential equations which is solved simultaneously to simulate the integrated system

$$\Delta \mathbf{A}(\mathbf{H}) \frac{d\mathbf{H}}{dt} = \mathbf{Q}(\mathbf{H}) + \mathbf{S} \qquad (10)$$

in which $\mathbf{H}$ = vector containing the water heads of all 2D cells, canal segments, and lakes together. The first term of Eq. (10) is derived from the first term of Eq. (9) which is equal to $\partial \mathbf{V}/\partial t$ in which $\mathbf{V}$ = volumes of water contained in water bodies. In the attempt to describe $\mathbf{V}$ as a function of $\mathbf{H}$, a new function $f_{sv}(H)$ referred to as the stage–volume (SV) relationship function is introduced. It holds behaviors very important to land surface features such as ridges and sloughs of South Florida and helps to determine water levels accurately. For each water body i = 1, 2, ... the function takes the form $V_i = f_{sv}(H_i)$. The slope of this function is defined as

$$\Delta A_i(H_i) = \frac{\partial f_{sv}(H_i)}{\partial H_i} \qquad (11)$$

in which $\Delta A_i(H_i)$ = effective plan areas of water bodies i. For 2D open water flow, these are the cell areas. For groundwater, these are s_c times cell areas. $\Delta \mathbf{A}(\mathbf{H})$ in Eq. (10) is the diagonal matrix whose elements (i, i) are $\Delta A_i(H_i)$. The reverse relationship of the SV relationship is defined as $H_i = f_{vs}(V_i)$. Both forward and reverse functions are used in the model to conserve mass during the mapping between V_i and H_i.

The term $\mathbf{Q}(\mathbf{H})$ of Eq. (10) gives the net inflow rate to each water body due to the action of all the water movers in vector form. In order to solve this coupled nonlinear system Eq. (10),

water mover equations are linearized prior to assembly as a large system of linear equations. The linearized form for any water mover is

$$Q_r(\mathbf{H}) = k_0 + k_i H_i + k_j H_j \qquad (12)$$

in which $Q_r(\mathbf{H})$ = discharge rate through the water mover r. The water mover r moves water from water body i to water body j. Linearization is performed to determine values of k_0, k_i and k_j using partial differentiation or approximate methods depending on the type of the water mover. They are used to build the resistance matrix $\mathbf{M}(\mathbf{H})$ that can be viewed as a result of the linearization of $\mathbf{Q}(\mathbf{H})$ as $\mathbf{Q}(\mathbf{H}) = \mathbf{M}(\mathbf{H}) \cdot \mathbf{H}$. The ordinary differential Eq. (10) with linearized $\mathbf{Q}(\mathbf{H})$ are solved using a weighted implicit method. Lal (1998) used the following system of equations to solve Eq. (10):

$$[\Delta \mathbf{A} - \alpha \Delta t \mathbf{M}^{n+1}] \cdot \Delta \mathbf{H} = \Delta t[\mathbf{M}^n] \cdot \mathbf{H}^n + \Delta t[\alpha \mathbf{S}^{n+1} + (1 - \alpha)\mathbf{S}^n]$$

$$(13)$$

in which α = time weighting factor, assumed to be in the range 0.6–0.8 for most integrated models and close to 1.0 when nonlinearities are severe and the model show signs of instability. This equation takes into account the water balance of all the water bodies during the time interval between times t^n and t^{n+1}. Knowing the volumes of water $\mathbf{V}(\mathbf{H}^n) = \mathbf{f}_{sv}(\mathbf{H}^n)$ at time step t^n and $\Delta \mathbf{H}$, it is possible to compute $\mathbf{V}^{n+1}$ using

$$\mathbf{V}^{n+1} = \mathbf{V}^n + \Delta \mathbf{A} \cdot \Delta \mathbf{H} \qquad (14)$$

The new heads $\mathbf{H}^{n+1}$ at time step $n + 1$ are computed using the SV relationship $\mathbf{H}^{n+1} = \mathbf{f}_{vs}(\mathbf{V}^{n+1})$. Heads are used in the model only to compute the hydraulic driving forces in the water movers. Except during this conversion, the model equations can be explained as a system of mass balance equations.

When water budgets for the model are needed, the volumes of water passing between water bodies, $Q_r(\bar{\mathbf{H}})\Delta t$ are computed first using Eq. (12). In this expression, the heads $\bar{\mathbf{H}}$ used are defined to be at a time $t^n + \alpha \Delta t$ and are computed as $\bar{\mathbf{H}} = \mathbf{H}^n + \alpha \Delta \mathbf{H}$. The water balance in any water body i can be verified for accuracy by comparing the change in water volume in the water body $\Delta A_i(H_i^{n+1} - H_i^n)$ with the summation of water mover discharges $Q_r(\bar{\mathbf{H}})\Delta t$.

Object Design

The process of abstraction and determining the relationship between abstractions form the basis for OO design. The basic abstractions used in the RSM model include: (1) "water bodies" that represent discretized cell elements, canal segments, and lakes which store water; (2) "water movers" that represent the only mechanisms to move water between water bodies; (3) "stage–volume relationship functions" that map between the stages and the volumes in water bodies; and (4) "pseudocells" that capture the local hydrologic function in the water bodies and compute their recharge. The entire hydrologic system can be decomposed into these and other abstract types such as transmissivity and conveyance functions $T(H)$ and $C(H)$. Fig. 1 shows some of the basic building blocks of the model. These abstractions allow a single numerical scheme to be used for the governing equations describing all flow types.

Abstract data types or classes in the model can be related to other classes through inheritance. A "subclass" or a "derived
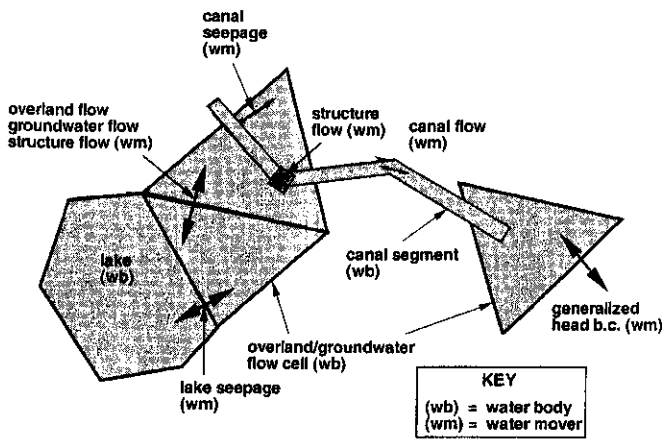
Fig. 1. Objects used in model

class" of a water body base class, for example, can be a discretized overland flow cell, canal segment, or a lake. They all inherit properties of the base class. Inheritance makes it possible to use polymorphism in OO modeling and allows functions to behave correctly depending on the object type. Polymorphism also allows water bodies to transform into canals, cells, and lakes while water movers can transform into canal flow, overland flow, and structure flow. Special methods associated with these objects fill the proper elements in the matrix. Four of the abstract classes used in the model are described below.

Water Bodies

The water body is the basic abstraction that collects water conservatively. Water body objects represent control volumes of the finite volume method, and provide a protected status for conservative variables such as water mass and solute mass. Cell elements, canal segments, and lakes become polymorphic water bodies. The head of the water body when needed is computed by calling the stage-volume relationship function of the water body described as $H_i = f_{vs}(V_i)$, $i=1,2,\ldots$. The first terms of Eqs. (9) and (10) represent change of volume in water bodies. Figs. 1 and 2 show examples of water bodies. Fig. 3 shows part of a class
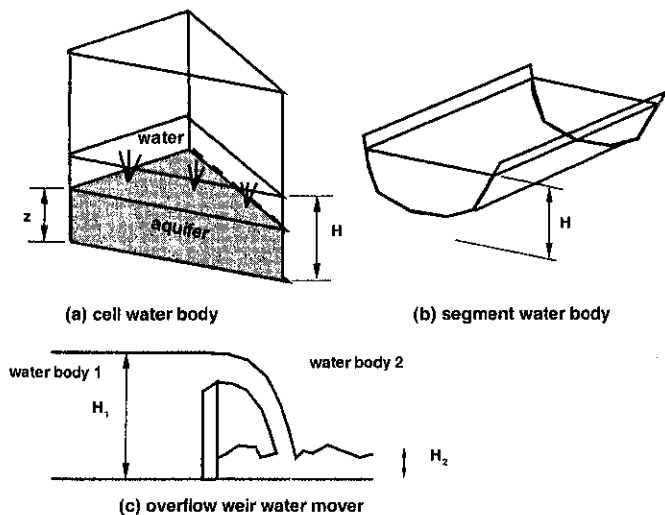

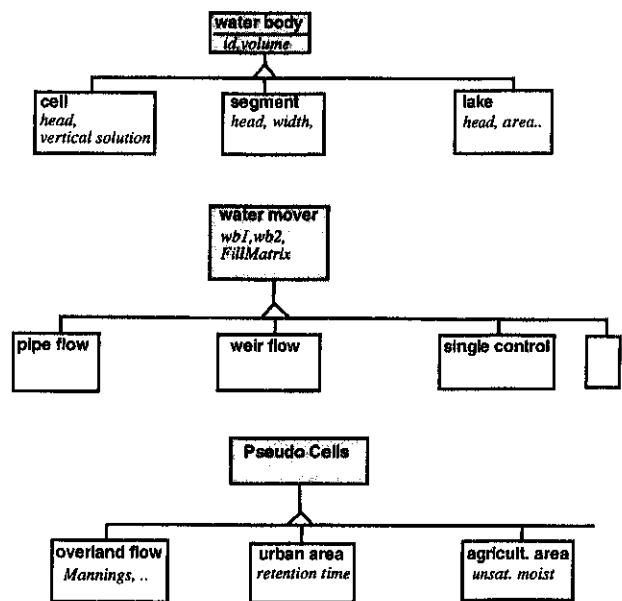**Fig. 2.** Examples of water bodies and water mover

Fig. 3. Class diagrams showing basic building blocks of model

diagram for water bodies which was written using the convention of Rumbaugh et al. (1991).

Water Movers

The water mover is the basic abstraction needed to transfer water between any two complex water bodies. It represents the flux term in the finite volume method, and transfer flow across control surfaces as in canal flow, overland flow, and all other kinds of flow such as structure flow. By design, they conserve mass. Some water movers such as those for overland flow, groundwater flow, and canal flow are created based on cell and canal network geometry. Water movers such as structure water movers are added as needed using input data. When new structure types are needed, they are added only as water movers. All water mover objects are placed in object containers to be accessed easily using the C++ standard template library features (Stroustrup 2000). Figs. 1 and 2 show sketches of sample water movers. Fig. 3 shows part of a class diagram for water movers. Since only water movers can move water between water bodies, the model can track mass balance of the system at the highest level of abstraction.

Overland Flow Water Mover
When the water levels are above ground in adjacent cells, overland flow takes place. The discharge in the water mover Q_r between two adjacent cells as described in Eq. (12) is computed using the circumcenter method derived for mixed finite elements (Lal 1998)

$$Q_r = \Delta l T_r \frac{H_m - H_n}{\Delta d_{mn}} \begin{cases} \text{for } H_m > H_n \text{ and } H_m > z_m \text{ and } H_m > z_n \\ \text{or } H_n > H_m \text{ and } H_n > z_n \text{ and } H_n > z_m \end{cases}$$

(15)

in which, H_m, H_n = water levels in triangular cells m and n; d_{mn} = distance between circumcenters of triangles m and n; Δl = length of the wall; z_m, z_n = ground elevations of cells m and n; and T_r = equivalent inter block transmissivity in the overland flow layer, computed based on the assumption that transmissivity varies linearly between circumcenters (Goode and Appel 1992; McDonald and Harbough 1988). Variable T_r is computed as

$$T_r = \frac{T_m + T_n}{2} \quad \text{for } 0.995 \leqslant \frac{T_m}{T_n} \leqslant 1.005 \tag{16}$$

$$T_r = \frac{T_m - T_n}{\ln \dfrac{T_m}{T_n}} \quad \text{otherwise} \tag{17}$$

T_m and $T_n =$ values for the cells defined in Eq. (2) for overland flow. Matrix elements filled up by the overland flow water movers are described in the paper by Lal (1998).

Groundwater Flow Water Mover

When simulating groundwater flow, transmissivity is assumed as constant inside a cell. The discharge in the water mover Q_r is computed using

$$Q_r = \Delta l \frac{H_m - H_n}{\left(\dfrac{l_m}{T_m} + \dfrac{l_n}{T_n} \right)} \tag{18}$$

in which l_m and $l_n =$ distances from the circumcenters to the wall; and T_m and $T_n =$ transmissivities described in Eq. (2).

Canal Flow Water Mover

When simulating canal flow, a linearly varying conveyance is assumed between canal segments. The equation for discharge between two segments m and n is the same as Eq. (15). The value of T_m, for example, for segment m is

$$T_m = \frac{A_m}{l_m \sqrt{S_n} n_b} \left(\frac{A_m}{P_m} \right)^{5/3} \tag{19}$$

in which $A_m =$ average canal cross sectional area of segment m; $P_m =$ average wetted perimeter; $n_b =$ average Manning roughness coefficient; and $l_m =$ length of a canal segment. When simulating canal networks, each pair of segments of a canal joint is considered as a canal water mover. A canal joint with n limbs has $n(n-1)/2$ canal water movers as a result. All these movers have to be considered before populating the matrix. Their summation computes the actual discharge.

Canal Seepage Water Mover

Seepage between a canal segment and a cell is described using a canal seepage water mover. The seepage rate q_l per unit length of the canal is derived using Darcy's equation

$$q_l = k_m p \frac{\Delta H}{\delta} \tag{20}$$

in which $k_m =$ sediment layer conductivity; $p =$ perimeter of the canal subjected to seepage; $\delta =$ sediment thickness; and $\Delta H =$ head drop across the sediment layer.

Canal overbank flow also occurs between a canal segment and a segment, but only when the cell has overland flow. The water mover used for this type of exchange is based on a simple broad-crested weir.

Structure Flow Water Mover

Linearization of structure equations to fit to the format of Eq. (12) is not always easy for most of the structures. Consider a structure whose discharge can be expressed as $Q_s = Q_s(H_u, H_d, G)$ in which H_u, $H_d =$ upstream and downstream water levels and $G =$ gate

opening. One of the simpler methods uses a previous call to function $Q(H_u, H_d, G)$ to obtain the following approximate linearization:

$$Q(H_u, H_d) = \frac{Q(H_u^n, H_d^n, G^n)}{H_u^n - H_d^n} (H_u - H_d) \tag{21}$$

in which the superscript n represents values from previous time steps. This linearization is accurate only in gradually varied flow, and works for a limited number of cases. Considering that $Q_s = Q_s(H_u, H_d, G)$ can be extremely nonlinear, differential equations with structure equations can be stiff and difficult to solve without special methods or small time steps. One and two-dimensional lookup tables and regression equations are also useful in describing structure flow.

Stage-Volume Relationships

Stage–volume relationship functions make it possible to use local stage–storage characteristics in integrated models. This feature is useful when the local topography is complex as in the case of ridge-and-slough formations in the Everglades, or as in the case of special land surface characterizations in agricultural and urban areas. The SV relationships can provide accurate water levels when the volume in a water body is known and vice versa. In the case of canals, they are used to obtain the water level when the canal properties are known. Local topography, storage coefficient, and other geometric information are used in the development of SV functions for 2D cells. They can be monotonically increasing complex functions or lookup tables based on experimental data. Some simple examples of $f_{sv}(H)$ are described below.

Stage-Volume Relationship Function $f_{sv}(H)$ for Cell with Horizontal Ground Surface

When the ground level is assumed horizontal, the SV relationship for a cell with a single layered aquifer is given by

$$V = f_{sv}(H) = A s_c (H - z_b) \quad \text{for } H < z \tag{22}$$

$$V = f_{sv}(H) = A s_c (z - z_b) + A(H - z) \quad \text{for } H \geqslant z \tag{23}$$

in which $z_b =$ elevation at the bottom of the aquifer; $z =$ elevation of the ground; and $A =$ cell area.

Inverse Relationship $f_{sv}(H)$ for Cells with Horizontal Ground Surface

Since the expression for flat ground is linear, Eqs. (22) and (23) can be used to obtain the following relationships:

$$H = f_{vs}(V) = z + \left\{ \frac{V}{A} - s_c (z - z_b) \right\} \quad \text{for } V > A s_c (z - z_b) \tag{24}$$

$$H = f_{vs}(V) = z_b \quad \text{for } V < 0 \tag{25}$$

$$H = f_{vs}(V) = z + \frac{V}{A s_c} \quad \text{otherwise} \tag{26}$$

Stage-Volume Relationship $f_{sv}(H)$ for Canal Segment with Rectangular Section

For a canal with a rectangular cross section, the relationship is

$$V = f_{sv}(H) = 0 \quad \text{for } H < z_c \tag{27}$$

$$V = f_{sv}(H) = BL(H - z_c) \quad \text{for } H \geqslant z_c \qquad (28)$$

in which z_c=elevation of canal bottom; L=length of canal segment; and B=canal width. The inverse relationships of most of the functions are complex and are not described here.

Pseudocells

The hydrologic system of South Florida covers areas with many types of land use. Most areas along the east coast are heavily urbanized, while some of the areas in the south are natural and wetland type. Areas south of Lake Okeechobee are mostly agricultural. Pseudocells are used to separate the complexities of the unsaturated subsurface flow accounting, irrigation practices, urban detention, and routing practices of the natural and managed systems from the regional system. Contribution of recharge from the local system to the regional system is computed using the mass balance condition given in Eq. (7). Pseudocells contain storage, routing, and simple management based interchange mechanisms to simulate infiltration, percolation, seepage, and urban drainage among other things. The *AFSIRS* model (Smajstria 1990), *CASCADE* model (SFWMD 2001), *NAM* model (DHI 1998), and a few other models are available as pseudocell models for HSE. The simplest pseudocell is one for open water where the equation for recharge Eq. (7) becomes

$$R = P - E \qquad (29)$$

Boundary Conditions

When solving 1D and 2D diffusion flow equations, only one boundary condition of discharge type or water level type is needed at each boundary. General head, uniform flow, and lookup table type boundary conditions are very useful with diffusion flow. These and other 2D boundary condition types are described in the paper by Lal (1998).

Some of the boundary conditions are simple enough that they can be applied to general water bodies. The flow boundary condition for a water body is one of them. It specifies the flow rate to a water body as

$$Q_i(t) = Q_B(t) \qquad (30)$$

in which $Q_i(t)$=inflow rate to water body i and $Q_B(t)$=specified inflow rate. Similarly, the head boundary condition for a water body states that the water level can be assigned to a constant value or a time series value. For a water body i, it is stated as

$$H_i(t) = H_B(t) \qquad (31)$$

in which $H_i(t)$=head at water body i at time t and $H_B(t)$ =assigned value. One problem of the head boundary condition (BC) is that it reorganizes the entire matrix and resets data in the contributing water movers, ruining the ability to compute water budgets. To avoid this, the head boundary condition can be applied to cell walls instead of cells and canal joints instead of canal segments. To apply a head BC at a cell wall, a certain discharge q_i is added to the cell i to bring about a change in wall head to $H_B(t)$. The discharge added is

$$q_i = \frac{T(H)l}{l_c}(H_B(t) - H_i) \qquad (32)$$

in which $T(H)$=transmissivity; H_i=cell head; l=wall length; and l_c=distance from the wall to the circumcenter. The general head boundary condition is also useful under certain groundwater conditions. It is described using the equation

$$q_i = K_G l(H_B(t) - H_w) \qquad (33)$$

in which K_G=specified conductance value in m/s; and H_w=wall boundary head. The uniform flow boundary condition is similar, and relates slope to flow rate q_i as

$$q_i = T(H)S_b \qquad (34)$$

in which S_b=slope of uniform flow associated with the water body.

Operation of Structures and Pumps

Some structures and pumps in the model are operated to achieve certain performance goals in the hydrologic system. The operations are based on rules assigned by water managers. Some of the operations are manual and others are automatic. Some operational rules are very complex because they have evolved over time based on historic events, human needs, and prior experiences of the water managers. The complex operational rules and logical directives applied to the structures and pumps are implemented as on/off type or proportional type functions on structures and pumps.

The purpose of operating structures and pumps is to achieve a desired performance in the system at the desired time. Some performances and conditions are mandated by legislation. A number of the performance measures used for South Florida are described in a comprehensive review study report (USACOE 1999). Currently many of the operations are rule based, and not necessarily optimal. However optimization can be used in the future to determine decision variables in the system. The methods available includenonlinear optimization methods (Brdys and Ulanicki 1994), optimal control methods (Gelb 1974), linear programming (Loucks et al. 1981), and optimization by simulation. These methods are built into the MSE section of the model. The methods currently used are described in the South Florida Water Management Model documentation (SFWMD 2001).

Water Budgets

In a compartmentalized landscape such as South Florida, determination of the water budget within a hydrologic basin or a compartment can be very important for many water managers. Water bodies and water movers are ideally suited to carry out water budgetcomputations because they track the volume of water contained in and passing through them using the abstract base class design. Each water body has an attached list of water movers that can report the discharges. The discharge computation for the water movers is carried out using Eq. (12) and the updated values of head and k. Volume changes are calculated using Eq. (14). An example showing the water budgets of two arbitrary water bodies is presented in Table 1. In the table, the lists of water movers attached to each water body, and the discharges in them are shown.

Model Errors

Even if mass balance errors in the finite volume method are small as shown in Table 1, other computational errors can be large depending on the discretization. Improper selection of spatial and temporal discretizations can make a model implementation ineffective in producing solutions of certain necessary scales. Recent

Table 1. Sample Water Budgets for Two Water Bodies on December 31 1992 in m³/day

Water body	Attached water movers	Inflow volume
Cell 192	Overland from cell 191	0.00
	Overland from cell 94	0.00
	Groundwater from cell 191	6.56
	Groundwater from cell 193	3,178.24
	Groundwater from cell 94	−44.39
	Seepage from segment 10008	−4,722.00
	Change in storage	−1,268.68
	Mass balance error	2.2×10^{-5}
Segment 10008	Flow from segment 10007	−63,347.40
	Flow from segment 10009	51,222.50
	Flow in weir	0.00
	Flow in bleeder	0.00
	Overbank flow from cell 191	6,436.07
	Seepage from cell 191	4,722.06
	Seepage from cell 96	65.60
	Seepage from cell 94	35.96
	Seepage from cell 95	13.61
	Change in storage	−861.50
	Mass balance error	-8.3×10^{-5}

studies show that proper discretization can be based on rules derived using analytical equations for numerical error (Lal 2000). These rules, along with rules regarding the applicability of the approximations of St. Venant equations, and rules to control nonlinear instability are important in making sure that solutions to integrated models are accurate.

Numerical error analysis methods are presented partly to demonstrate their use in model verification, and partly to demonstrate how to control model errors. A numerical experiment is carried out using a sinusoidal boundary disturbance in a 2D groundwater model domain to demonstrate that numerical errors of the model agree with the analytical estimates. The experiment is also used to demonstrates how to calculate numerical errors for a model when spatial and temporal discretizations are known. Dimensionless spatial discretization $\phi = k\sqrt{\Delta A}$ is defined for triangles instead of $\phi = k\Delta x$ for rectangles in which k=wave number of the disturbance and ΔA=cell area. For the experiment, a confined groundwater domain is created in a 10×10 km area and populated with 3,200 approximately isometric triangles. A 1D sinusoidal head disturbance is then introduced into one of the boundaries, and water heads at different distances away from the wall are then monitored over long periods. The heads are compared with the analytical estimate to obtain observed errors in the amplitude. The observed errors are also compared with analytical error estimates (Lal 2000).

To demonstrate analytical error estimation, the analytical solution for 1D groundwater flow is first expressed as

$$H(x,t) = H_o e^{-kx} \sin(ft - kx) \qquad (35)$$

in which f=angular velocity or angular frequency of the boundary disturbance defined as $2\pi/T_p$ (rad/s); T_p=period of the disturbance; H_o=amplitude of the disturbance; x=distance from the boundary; and $k=\sqrt{fs_c/(2T)}$. The analytical solution for the numerical error for this problem is (Lal 2000)

Fig. 5. Contours of ε (%) against ϕ and ψ

$$\varepsilon_T(x) = \frac{k\varepsilon}{\beta\phi^2}x = \frac{2k\varepsilon}{\psi}x \qquad (36)$$

in which $\varepsilon_T(x)$=maximum error over the duration as a fraction of the amplitude at a distance x; Δt=time step; $\psi = f\Delta t = (2\pi/T_p)\Delta t$ =dimensionless time step; and ε=maximum error per time step as a fraction of the amplitude; $\beta = T\Delta t/(s_c\Delta A)$ in which β=useful dimensionless parameter which has to be less than 0.5 for the stability of the explicit method when solving the 1D groundwater flow equation. It can be shown by substitution that $\beta = \psi/(2\phi^2)$ for this problem. Analytical values of ε (or ε_{anal}) can be obtained as a function of β and ψ or ϕ and ψ (Lal 2000). Fig. 5 is a contour plot of the latter.

In order to show that analytical estimates of numerical error ε_{anal} compare well with measured model error ε_{obs}, values of ε_{obs} are obtained first. To do this, $\varepsilon_T(x)$ of Eq. (36) are plotted against x for a number of model runs and Eq. (36) is fitted to determine the slope. Fig. 4 shows one such plot when the period T_p=8.6 h or $f=2\pi/T_p=2.029 \times 10^{-4}$ s^{-1}, Δt=30 min, $\sqrt{\Delta A}$=152 m for themesh, T=20.0 m²/s (for an arbitrary porous medium), and s_c =0.2 confirming that the error behavior is linear as shown in Eq.
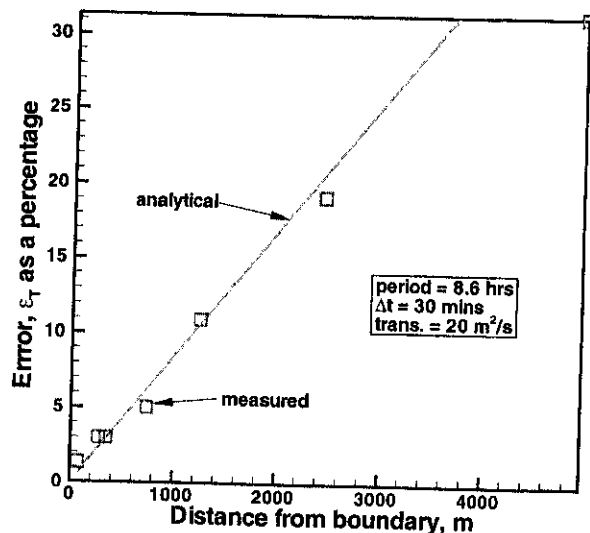

Fig. 4. Variation of percentage error with distance

Table 2. Model Runs Used in Comparison of Analytical and Observed Errors

T_p	Δt	T (m^2/s)	k (m^{-1})	ϕ	ψ	β	ε_{anal} (%)	ε_{obs} (%)
8.6 h	0.5 h	20.0	0.00101	0.153	0.365	7.791	1.53	1.56
8.6 h	0.5 h	2.0	0.00318	0.484	0.365	0.779	1.81	1.57
5.1 h	0.5 h	20.0	0.00131	0.199	0.616	7.791	4.1	9.0
20.0 days	1.0 day	2.0	0.00043	0.0648	0.314	37.39	1.13	1.12

(36). Using the slope of the graph $2k\varepsilon/\psi = 8.65 \times 10^{-5}$, the value of ε_{obs} can be obtained as 1.53% using $k = \sqrt{fs_c/(2T)} = 0.001007$, and $\psi = 0.365$. The analytical value $\varepsilon_{anal} = 1.57\%$ can be obtained using the method by Lal (2000) or Fig. 5 with $\phi = k\sqrt{\Delta A} = 0.153$ and $\psi = 0.365$. The observed value of ε_{obs} for the model shown as a dot in Fig. 5 compares well with the analytical values in contours. Table 2 shows the summary of the model runs shown in Fig. 5.

The analytical values of ε_{anal} in Fig. 5 can be used to predict model errors in any future model when the cell size ΔA and the time step are known, and when the stress comes from a similar forcing function. To derive an equation for maximum absolute error ε_{abs} in the domain in units of length, Eqs. (35) and (36) can be used

$$\varepsilon_{abs} = \frac{2H_0\varepsilon}{e\psi} \tag{37}$$

where $e = 2.718$. The maximum error is negative, and occurs at a distance $x = 1/k$ from the boundary during the peak and the trough of the cycle. The steps involved in computing the error are: (1) compute $f = 2\pi/T_p$ knowing the period of the water level disturbance; (2) compute $\psi = f\Delta t$; (3) compute $k = \sqrt{fs_c/(2T)}$; (4) compute $\phi = k\sqrt{\Delta A}$; (5) use Fig. 5 to read ε for the ϕ and ψ; and (6) compute ε_{abs} using Eq. (37). For the problem with $T_p = 8.6$ h described earlier, the error is $2.0 \times 0.0157/(2.718 \times 0.365) = 3.2\%$ of the disturbing amplitude according to Eq. (37). If this error is too large, finer discretizations have to be selected.

Model Verification and Applications

The computational methods used in the RSM model have been verified in the past using a number of methods (Lal 1998). The most rigorous verification of the model was carried out using an analytical solution for the problem of stream–aquifer interaction (Lal 2001). In the test, sinusoidal water level disturbances of varying frequency were used to disturb a canal interacting with an aquifer. The decay and the delay of the solution for water levels in the system were obtained using the model and the analytical method. The results of the test plotted using dimensionless variables show that the numerical solution agrees with the analytical result. The error analysis described in the previous section is also useful as a verification method.

The earliest application of the *FORTRAN* version of the model with 2D overland and groundwater capabilities was on the Kissimmee River (Lal 1998). The C++ version of the model with 2D capabilities was used to simulate flows in the Everglades National Park by Lal et al. (1998). The current application uses the fully integrated model in C++. The L-8 basin of South Florida was selected to demonstrate the complexity of the problem, while keeping the presentation simple. Many of the hydrologic components and issues related to the L-8 basin are typical for South Florida. Although it is usually not easy to isolate individual basins in the South Florida hydrologic system, the L-8 basin is relatively isolated and therefore somewhat easy to study. Fig. 6 shows a site map of the L-8 basin. The details listed below will demonstrate how even the simplest basins can have a large amount of critical information to consider. As it can be seen, many of the complexities are due to human influences.

Brief Description of L-8 Application

The L-8 basin is located within an area approximately 100 km $\times$ 100 km in size, near the northern boundary of Palm Beach County, Fla. It is delineated by artificial levees, and consists of natural, agricultural, and urban areas adjacent to each other. The basin is bounded by the L-8 canal and the M-canals on the south, the Pratt and Whittney Complex and the Indian Trail Drainage District on the east, and the Lake Okeechobee on the west. It includes the Corbett and Dupuis Wildlife areas to the north, parts of the Village of Royal Palm Beach (VRPB) to the east, an agricultural area covering citrus to the south, and an agricultural area to the north. Water supply needs of the system include the agricultural demand in the north drawn from the L-8 canal, irrigation withdrawals along the M-canal and water withdrawals from M canal for the City of Palm Beach Utilities Department. Eastward water movement along the M-0 canal is due to the pump station on the canal. The capacity of the L-8 canal is about 14 m^3/s when its water level is about 4.6 m above sea level. The capacity of the M-canal is about 8.5 m^3/s. The M-1 canal drains some of the water in the Village of Royal Palm Beach to C-51 canal.

The L-8 canal is connected to Lake Okeechobee at culvert S10-A at the north end. During the simulation period, excess runoff from L-8 is routed to the lake by gravity during flooding. At the southern end, L-8 is connected to the structure complex S-5A, which is capable of sending water to the south, L-8 canal, or the
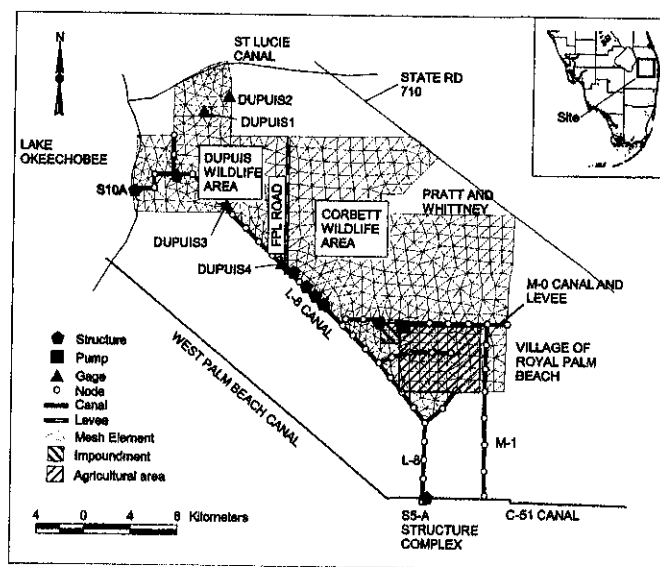

Fig. 6. Site map of L-8 basin

C-51 canal depending on a number of conditions. The model uses a head boundary condition at culvert S10-A and a discharge boundary condition near the structure S5-A. The L-8 canal and all the other canals are fully integrated with the 2D flow domain except near S5-A where the canal runs by itself. Most of the 2D domain covered in the model is assumed to have no-flow boundaries. The boundary condition near the agricultural area south of S10-A is a constant wall head boundary set to 4.3 m.

A number of levees restrict overland flow in the basin. The levee along the L-8 canal is the most prominent, and prevents water in the Dupuis and Corbett wildlife areas from directly entering into the L-8 canal. There are four water control structures and bleeders located in the levee to maintain the water levels in the 5.2–5.8 m range and release the excess to L-8. A second levee, marked as FPL road on the map, runs from north to South between the Dupuis and Corbett areas preventing overland flow between them. A third levee prevents overland flow from the northeastern quarter entering the VRPB except through a culvert structure. The remaining canal sections in the southwestern quarter are assumed to be without levees and are therefore subjected to both stream–aquifer and stream–overland flow interactions.

The operation of the 292 ha impoundment by the Indian Trail Water Conservation District (ITWCD) is used to demonstrate how one of the water management operations is simulated in the model. The impoundment is used during floods to maintain low water levels in areas draining to M-0 canal. The ITWCD operates pumps sending water into the impoundment when the flood levels at the M-0 canal exceed critical levels. The pump capacity is 31 m³/s. The outflow from the impoundment passes through three outflow structures with 1.4 m discharge pipes and a 6.4 m invert elevation. The discharges in the pumps can be characterized approximately using a 1D lookup table in XML similar to the following. The following is also a self explanatory example of the use of XML in data entry.

```
〈single_control wmID="3" id1="10034" id2="354"
     control="10034" revflow="yes"
     label="Pump at the impoundment"〉

     5.0   0.0
     5.1   6.3
     5.3  12.6
     5.9  12.6
     6.1  31.0

〈/single_control〉
```

The lookup table moves water from a water body with ID =10034 which is the canal segment in M-0 to water body with ID=354 which is a cell in the impoundment. Pumping is controlled by water level in the water body 10034. When the water level of the control is 5.9 m, the discharge rate is 25.2 m³/s as shown in the lookup table. The identification tag of the water mover is 3, which is used when assigning an operational logic. The operational logic is written at a separate section of the model called the management simulation engine not described here.

The total simulation period used is between 1992 and 1995, while 1994 is used for calibration. The time step selected is 1 day because of data availability. The area is discretized using 1,027 cells and 49 canal segments. Figs. 6 and 7 show the discretizations. Daily rainfall and potential evapotranspiration are provided to the model in a 3.22×3.22 km square mesh. The first seven months of the simulation are used for initialization. Fig. 7 shows the water levels and the water velocity vectors one year after the simulation has begun. The figure shows the drainage patterns in
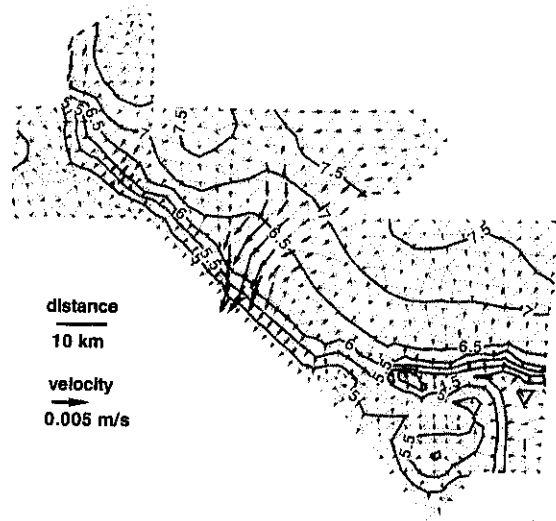

Fig. 7. Water levels and flow velocities of L-8 in January, 1993

the L-8 basin and the confluence of flow into the area where the structures are located. The flow is prevented from moving into Dupuis because of the levee between Dupuis and Corbett. Fig. 8 shows water levels at gages marked DUPUIS1 and DUPUIS2 in the northern part of the basin. Fig. 9 shows that water levels in the basin very close to the canal are influenced by the canal levels. The correlation coefficients for DUPUIS1-4 gages calculated according to Flavelle (1992) are 0.80, 0.81, 0.85, and 0.88 and the standard error estimates are 0.03, 0.03, 0.15, and 0.13 m, respectively. All the water levels indicate that the model is capable of representing the system reasonably under the natural stresses of the rain and imposed stresses of the L-8 canal. Fig. 10 shows the simulated and computed discharges in the L-8 canal for the same period.

The finite volume method and the object oriented code design are responsible for some of the water budget functions of the code. These capabilities are illustrated using water budgets of two water bodies for one day of the simulation period. Table 2 shows the water budgets of cell 192 and segment 10008 on December 31, 1992.
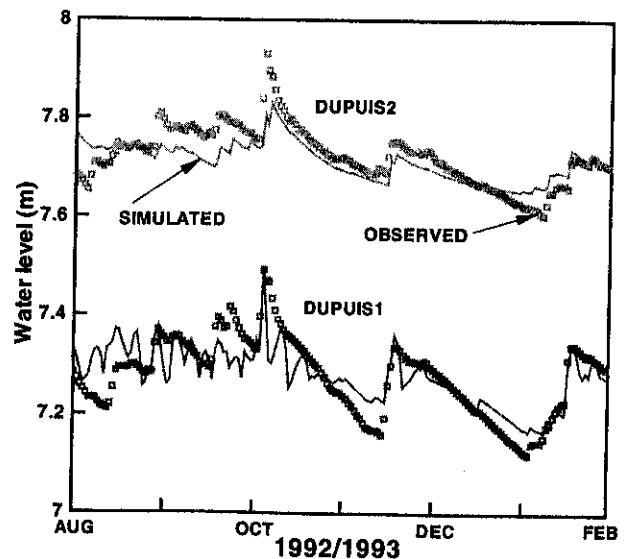

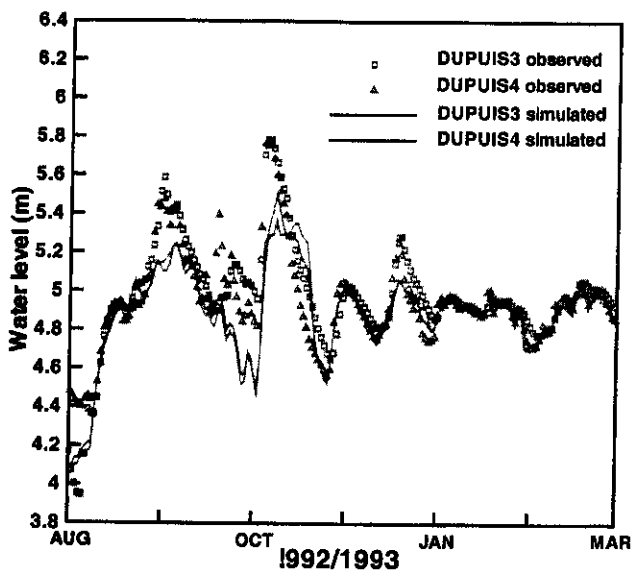**Fig. 8.** Water levels in Dupuis gages 1 and 2

Fig. 9. Water levels in Dupuis gages 3 and 4

Current and Future Applications

One of the functions of RSM in South Florida is to serve as a regional hydrologic model. Since RSM provides support to accommodate complex site-specific conditions using features such as pseudocells, SV relations, transmisivity functions, and conveyance functions, it is possible to design many complex model applications without making code changes. Scientists familiar with complex local conditions and management rules can develop pseudo cell model objects describing local agricultural practices, permitted rules and other informations. Current areas of application of the model include the Water Conservation Area 1 (16,292 cells), South West Florida (40,000 cells), Loxahatchee River watershed (7,247 cells), Welter (2002), Southern Everglades (52,817 cells) in Florida, the Kala-Oya basin in Sri Lanka (3,200 cells), Lal et al. (2004), and the South Florida Regional Simulation Model (SFRSM).
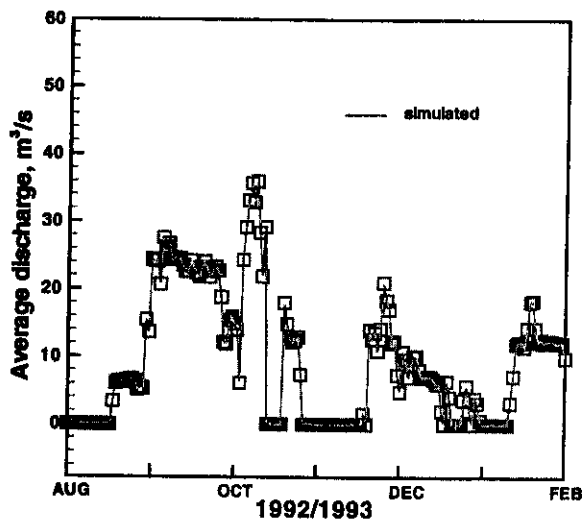

Fig. 10. Discharges at S-10A

Summary and Conclusions

An implicit finite volume method, a high-speed sparse solver, and the object oriented design approach contributed to the development of a fully integrated regional hydrologic model. A number of abstractions such as the water body, water mover, SV relationship, and pseudocells were used to accommodate the complex hydrologic features of the system seamlessly into one simple computational algorithm. An object oriented design provided an unlimited capability for the model to expand. The implicit method helped to make it stable.

The model was applied to a small but complex hydrologic basin in South Florida to demonstrate how different hydrologic components with different land use types could be incorporated into one model application. Results show that the model is capable of simulating the water levels and discharges observed in the field. Results also show that the model can provide consistent water budget information for model components.

The limited error analysis shows that the numerical errors of the model results agree with the error estimates computed using analytical methods developed by Lal (2000). Analytical estimates of numerical error are extremely useful in designing suitable model discretizations with known numerical error limits.

Acknowlegments

Notation

The following symbols were used in this paper:

A_c = cross sectional area of canal;
A_l = lake area;
B = width of canal segment;
$C(H)$ = conveyance of overland flow;
f = frequency (rad/s);
$f_{sv}(H)$ = stage–volume relationship function converting head to volume;
H = water head;
$\mathbf{H}^n$ = heads of water bodies at time step n;
k = wave number defined as $2\pi/$wavelength;
$\mathbf{M}$ = resistance matrix;
n_b = Manning coefficient;
R = hydraulic radius of canal;
R_{rchg} = net contribution to recharge from local hydrology to regional system;
S = water surface slope;
s_c = storage coefficient of soil;
$T(H)$ = transmissivity of aquifer;
t = time;
U, F, G = conservative variables in equations of mass balance;
V = volumes of water contained in water bodies;
W = source and sink terms in continuity equation;
x, y = Cartesian coordinates;

z_m = ground elevation of cell m;
α = time weighting factor; and
$\Delta\mathbf{A}$ = diagonal matrix of effective areas.

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